Date Planned : __ / __ / __	Daily Tutorial Sheet – 14	Expected Duration : 90 Min
Actual Date of Attempt : __ / __ / __	Level – 3	Exact Duration :________

162. If in a triangle, s denotes the semi-perimeter and a,b,c denote the lengths of sides, then prove that

$$\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix} = 2s^3(s-a)(s-b)(s-c)$$

163. Prove that $\begin{vmatrix} ax-by-cz & ay+bx & cx+az \\ ay+bx & by-cz-ax & bz+cy \\ cx+az & bz+cy & cz-ax-by \end{vmatrix} = \left(x^2+y^2+z^2\right)\left(a^2+b^2+c^2\right)(ax+by+cz)$

164. If $\Delta(x) = \begin{vmatrix} a_1+x & b_1+x & c_1+x \\ a_2+x & b_2+x & c_2+x \\ a_3+x & b_3+x & c_3+x \end{vmatrix}$, show that $\Delta''(x)=0$ and that $\Delta(x)=\Delta(0)+Sx$, where S denotes the sum of all the cofactors of all the elements in $\Delta(0)$.

165. If α,β,γ are different from 1 and are the roots of $ax^3+bx^2+cx+d=0$ and $(\beta-\gamma)(\gamma-\alpha)(\alpha-\beta)=\frac{25}{2}$, then prove that $\begin{vmatrix} \frac{\alpha}{1-\alpha} & \frac{\beta}{1-\beta} & \frac{\gamma}{1-\gamma} \\ \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \end{vmatrix} = \frac{25d}{2(a+b+c+d)}$

166. If $X = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, then prove that $(pI+qX)^m = p^m I + mp^{m-1}qX$, $\forall p,q \in R$, where I is a two –rowed unit matrix and $m \in N$.

167. If A is an upper triangular matrix of order $n \times n$ and B is a lower triangular matrix of order $n \times n$, then prove that $(A'+B)\times(A+B')$ will be a diagonal matrix of order $n \times n$ [assume all elements of A and B to be non-negative and an element of $(A'+B)\times(A+B')$ as C_{ij}].